MATHEMATTIC SOLUTIONS

Statements of the problems in this section originally appear in 2021: 47(1), p. 4–6.

MA101. Standard six-sided dice have their dots arranged so that the opposite faces add up to 7. If 27 standard dice are arranged in a $3 \times 3 \times 3$ cube on a solid table what is the maximum number of dots that can be seen from one position?

Originally problem I21 from the 2014 competition of Australian Math Trust.

We received 5 submissions, of which 4 were correct and complete. We present the solution by the Missouri State University Problem Solving Group.

More generally, we will answer the analogous question for abc standard dice arranged in an $a \times b \times c$ cuboid. The maximum number of faces that can be seen are three faces that share a common vertex. There are

$$(a-1)(b-1) + (a-1)(c-1) + (b-1)(c-1)$$

cubes with one face showing, $(a-1)+(b-1)+(c-1)$ cubes with two faces showing, and one cube with three faces showing. This gives

$$\begin{aligned} &6\left((a-1)(b-1) + (a-1)(c-1) + (b-1)(c-1)\right) \\ &+ (6+5)\left((a-1) + (b-1) + (c-1)\right) + (6+5+4) \\ &= 6(ab+ac+bc) - a - b - c \end{aligned}$$

as the maximum number of dots. For $a = b = c = 3$, we have 153 dots. The figure below shows the case when $a = 3, b = 4$, and $c = 5$. The white cubes have one exterior face, the light gray cubes have two, and the dark gray cube has three.